Against Utility in School Mathematics and in Educational Research: a voice from the Twilight Zone

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The bookshop near to my Institution used to be called 'The University Bookshop'. Entering through the main door, you will be confronted by two wide stairways. One goes down to the basement in which are housed atlases and books on medicine, the natural sciences, computer sciences, and mathematics; the foundation of human knowledge. The other stairway goes up to the first floor. Here, compact discs (classical and jazz only) are sold as well as books in the music department, there are dictionaries and encyclopaedias and massive tomes on art and architecture; a museum of human achievement. The ground floor itself houses the bulk sellers—popular and classical literature, magazines and 'bargain books'. A narrow staircase leads from the first to the second floor and another from the second to the third and top floor. If someone is coming down when you want to go up, you have to wait for them; these are very narrow stairs. Having reached the top floor, you turn left and keep going to the end of the building. There, at the farthest point from the main entrance, you will find the section on educational research. Curiously, if you turn right instead of left and go to the opposite end of the building, you will find books on sociology. An economic base, perhaps, interrogated with the precision of the natural sciences and supporting a cultural superstructure within which gravity decreases with altitude. Those of us who work in educational or sociological research are quite used to this marginalising of our activities to an intellectual twilight zone. The more so, those of us who think of ourselves as sociologists of education. Spurned by practitioners and educationalists as having nothing of any practical value to say, we are accused of wrapping-up our (generally left-wing) political agenda in impenetrable sociologese.

On a recent exploration of the bookshop, I visited the medical department, looking for a book on medical education. There is no section on medical education which would correspond to the mathematics and science education section in educational research. The book that I had been looking for was A Handbook for Medical Teachers (Newble & Cannon, 1994); I found it under 'general practice'. The foreword bemoans the general indifference to, even contempt for, pedagogy by medical and other academics. It recounts a story about a young assistant professor of mathematics at a 'leading research university' who won the 'best teacher' award one year. His department chairman summoned him and announced, '"You will win no 'brownie points' with me or this department for that kind of crap"' (p. ix). Ironically, the handbook reproduces itself as precisely 'that kind of crap'. It is in A4 format, with cartoons on the cover and as marginal illustrations throughout the text; books on endocrinology don't look like this.

Mathematics itself, of course, is securely located as highly valued knowledge. But its dismissal of lower status practices, including pedagogic practice, may prove its undoing. Consider the following anecdote from Mike Cooley:

> At one aircraft factory they engaged a team of four mathematicians, all of PhD level, to attempt to define in a programme a method of drawing the afterburner of a large jet engine. This was an extremely complex shape, which they attempted to define by using Coon's Patch Surface Definitions. They spent some two years dealing with this problem and could not find a satisfactory solution. When, however, they went to the experimental workshop of the aircraft factory, they found that a skilled sheet metal worker, together with a draughtsman had actually succeeded in drawing and making one of these. One of the mathematicians observed: The may have succeeded in making it but they didn't understand how they did it. Cooley, 1985; p. 171; my emphasis)

The opposition is, of course, one of social class, constituted in and by the intellectual/manual division of labour. The elitism displayed by the mathematician is hardly challenged by the 'manual' workers' success in the face of their own failure. Real success, it seems, lies in the solution of the theoretical problem which, alone, provides access to understanding and knowledge about the world.

In a political discourse which is informed by a left of centre rejection of the intellectual/manual hierarchy and which recognises an urgency in the matter of the practical reconstruction of society, the mathematician's elitism is anathema. Mathematics and science divorced from the 'real world' is unacceptable and educational organisation practice must be oriented towards the unification of this division. In his plenary lecture to last year's SAARMSE conference, Michael Young contrasted two approaches to the science curriculum.

> ... a Curriculum of the Past can be distinguished from a Curriculum of the Future. Science in a Curriculum of the Past is insulated as separate subjects—, divided— between academic science that is defined in terms of subjects or disciplines and vocational science defined in relation to specific job requirements, and past oriented— since criteria as to what counts as academic or vocational knowledge refer to past practice of subject specialists and craftsmen. Finally learner assessment in the Curriculum of the Past is designed to minimise risks and encourage memorising.
>
> In a Curriculum of the Future, science teachers would emphasise connective skills— how science is related to other knowledge areas and the whole curriculum, the integration of theory and practice— how scientific knowledge can inform practical outcomes, be future oriented by emphasising how what a student learned would help her or his future activities, and they would encourage risk taking to prepare students for uncertainty rather than certainty in their future lives. (Young, 1994; p. 553)

Within the context of a South Africa approaching its first democratic election, looking back on its divided past and forward to its united future, Young effectively associates the curriculum that he wants to oppose with apartheid. Nevertheless, it may be that the tension between division and unification is inevitable and, indeed, productive. Certainly is has been apparent at this year's conference. Jonathan Jansen, for example, argued for the need to bridge research and practice in education, whilst Olugbemiro Jegede and Mishak Ogunniyi both introduced the productive possibility of multiple world views. I want to explore this tension through, initially, a consideration of some secondary school mathematics texts and, subsequently, via a discussion of apprenticeship.

The analysis of school mathematics texts

In its policy document of education and training, the African National Congress adopted much the same line as Michael Young in suggesting that:

> ... science and mathematics education and training, both school-based and work-based, must be transformed from a focus on abstract theories and principles to a focus on the concrete application of theory to practice. It must ensure that students and workers engage with technology through linking the teaching of science and mathematics to the life experiences of the individual and the community. (ANC, 1994; p. 84)

It may well be thought that such a position is justified by work such as that of Jean Lave and colleagues (1984, 1988). These anthropologists have found that success in everyday practices in the US, such as supermarket shopping, owes very little to school mathematics. Furthermore, Brazilian street traders seem to be very effective in apparently 'mathematical' practices without ever having had much instruction at school (Carraher et al, 1985, 1993).

Looking at school textbooks, it is perhaps not difficult to see the basis for this division between the school and the 'real world'. Plate 1 shows a page from SMP Book G7. This is the seventh in a series of eight books for 'less able' students in the third to fifth years of secondary school. Prior to this, the text has introduced the 'rule' circumference = 3 X diameter. Here, it is pointed out that this is always an underestimate and we should add ten per cent 'to be on the safe side'. This addition makes the approximation an overestimate. As with all algorithms, this does not work in all contexts. There may well be cases when an underestimate rather than an overestimate may be the 'safe side'. However, I want to draw attention to the implausibility of the narratives that are used to constitute the tasks in this section. Firstly, we are asked to believe not only that Jim is stupid enough not to leave any spare tape for fraying and cutting errors, but that he actually manages to buy exactly 90 cm. Presumably, when he goes back to the shop after adding ten per cent to his original estimate, he asks for 99 cm (although the answer book does give 1 m as an alternative). In task C3, Zola is supposed to calculate the length of baking paper needed to line the tin. In fact, baking paper often comes on a roll of fixed width which would constitute the ‘length’; the length to be cut would depend upon the height of the tin and more than one length would be needed if the resulting strip was too short. Task C6 is on the following page:

> Eve packs rolls of carpet.
> When the carpet is rolled up
> its diameter is about 45 cm.
> Eve puts 3 bands of sticky tape
> round the roll.
> (a) About how much tape does she need for one band?
> (b) How much does she need for each roll of carpet?
> (c) The sticky tape comes in 100 m rolls.
> Roughly how may carpets can Eve pack with 1 roll of sticky tape?
> (G7, p. 19; marginal drawing omitted)

Eve apparently packs just one kind of carpet (different pile thicknesses would result in differently sized rolls) and uses just a single turn of tape for each band (the answer in the Teacher’s Guide gives ‘Accept 145-155 cm’ for part (a)) and there is no waste on the roll of tape.

This textbook and the others in the series present mathematics as entirely dedicated to the optimising of everyday practices. There is a virtual exclusion of mathematics for its own sake. Yet what is being presented is clearly a recontextualising of the everyday. In his plenary lecture, Olugbemiro Jegede described school science as a ‘mythic science’; here, school mathematics is mythologising domestic and workplace activities. Mundane practices are arranged, not as people actually act in concrete, everyday settings, but according to mathematical principles. It is as if mathematics were casting a gaze on people’s lives, reorganising them according to its own structures and then handing them back: you see how much better life would be if we were all mathematicians. But it wouldn’t be better, because mathematised solutions always fail to grasp the immediacies of the concrete settings within which, as Jean Lave points out, problems and solutions develop dialectically. The mathematical gaze generates a virtual reality, a ‘public domain’ within which all is rational and all is calculable.

The development of the geometric ratio, π, in SMP 11-16 Book Y1, chapter 7, looks very different. Book Y1 is the first in the series intended for the ‘top twenty per cent’ of the same target age-group as that of the G series. The Teacher’s Guide describes the Y1 chapter as follows:

> The idea of ratio, as developed in chapter 6, plays an important part in this chapter. The ratio $\frac{\text{circumference}}{\text{diameter}}$ in a circle is approached as a limiting case of the ratio $\frac{\text{perimeter}}{\text{diameter}}$ in a regular polygon.
> (Teacher's Guide to Book Y1, p. 30)

Unlike the G texts, the Y series is multiply and often explicitly recursive, so it is not generally possible to mark out the beginning of a particular topic in an unambiguous way. However, we can pick up the developing discourse on geometry at the start of section C of chapter 6, which introduces the specialised expressions, ‘scale factor’ and ‘enlargement’. Section E, in the same chapter, introduces the terms ‘similarity’ and ‘ratio’. What is being established, mathematically, is a relationship between the geometrical transformation, ‘enlargement’, the comparative term, ‘similar’, and geometrical ratios. This sets the basis for the following chapter, the opening of which is shown in Plate 2.

Chapter 7 opens with a formal definition of terms. The exposition also articulates with ‘enlargement’ and ‘similarity’ and with geometric ratios from the previous chapter. The ratio, $\frac{\text{perimeter}}{\text{diameter}}$, is initially referred to as the ‘p-number’ of a polygon and sections A and B of the chapter tabulate and graph p-numbers against number of sides up to 48; the graph is followed by an exposition:

> As the number of sides increases, the polygon looks more and more
> like a circle. The p-number gets closer and closer to the
> p-number of a circle, which is just over 3.14.
> [...]
> The p-number of a circle is denoted by the Greek letter p,
> which is written π and pronounced ‘pi’.
> The value of π, correct to 5 decimal places, is 3.14159.
> The perimeter of a circle is called the circumference of the circle.
> So the ratio $\frac{\text{circumference}}{\text{diameter}}$ is π. Or, in other words,
> π is the multiplier from diameter to circumference.
> [...]
> (Y1, pp. 90-1; indexical diagrams omitted)

The exposition gives a new conception of a circle as the limit of a series of polygons having increasing numbers of sides (the circle appeared earlier in the book as a locus). It also introduces the term ‘circumference’ and the expression π, which is described as both a ratio and as a multiplier. At the end of the chapter, several approximations to π are given including its decimal expansion to thirty-five decimal places. The Y text employs what I refer to as a ‘generalising strategy’, in which mathematical signs are articulated so that the general principles of mathematical practice are rendered more visible. The term ‘circle’ here denotes a mathematical object which is defined as the limiting value of a polygon as the number of sides increases. An extensive array of connotations has been attached to ‘circle’. This array includes: polygon, perimeter, side, circumference, radius, diameter, 3.14159, π, ratio, graph, $\frac{\text{circumference}}{\text{diameter}}$, multiplier, and (which is also implicated in the exposition). All of these terms have specialised mathematical meanings. The text articulates them in definitions, principles and propositions to form part of a complex which I refer to as the ‘esoteric domain’ of mathematical practice. Statements and propositions made within this domain acquire a high degree of context independence, because their meanings are effectively exhausted by explicit definitions. A ‘circle’ can be defined in a number of ways, but these are more or less consistent with each other, so that the meaning of ‘circle’ is very close to being complete. We know that a mathematical use of ‘circle’ could not refer to the group of our friends and acquaintances; mathematics itself determines meaning systematically and with a high degree of linguistic precision.

I distinguish this situation from that of everyday practices where meanings are far more context-dependent. Linguistic utterances catch at the everyday, but can never exhaust it. Even something as apparently definitive as a shopping list lacks precision, because so much depends upon what you find when you get to the market. In everyday practices, language itself is used much in the same way as physical resources: it is far less a matter of whether or not an utterance represents a true proposition or a linguistically well-formed statement than whether it serves the purpose within the immediate context. Esoteric domain mathematical practices, then, impose a regulation on language: they constitute a discourse; they establish what may or may not be said or written. Discourse may be described as saturating the practices in a way that it does not in the everyday. I refer to practices such as mathematics as exhibiting high ‘discursive saturation’ and those such as the everyday as low ‘discursive saturation’.

The relationship between the circumference and diameter of a circle is introduced in Book G7 almost entirely outside of the esoteric domain, but there are three instances of esoteric domain exposition. The first gives an algorithm for the calculation of radius from diameter, or vice versa, using the expression, ‘of’, in preference to the mathematically more usual, ‘X’:

> The diameter is the width of a circle.
> The distance from the centre of a circle
> to the edge is half the diameter.
> We call this distance the radius.
>
> radius = ½ of diameter
>
> (G7, p. 15; graphical index omitted; bold text in red in original)

The second provides the original algorithm for the calculation of circumference that was mentioned earlier. The equality symbol is used, even though the algorithm represents an approximation:

> The distance round the edge of
> a circle has a special name.
> It is called the circumference
> of the circle.
> The circumference is a bit longer than 3 times the diameter.
> If you only want a rough answer for the circumference of a circle
> you can use
>
> circumference = 3 X diameter

(G7, p. 16; bold text in red in original)

The final section of esoteric domain exposition gives the ‘safe side’ algorithm discussed above. In contrast to the Y1 text, there is no generalising of mathematical signs, so that the principles of the esoteric domain remain invisible. Mathematics is presented more or less as a discrete set of facts rendered as algorithms. For ‘less able’ students, then, even esoteric domain mathematics is degraded; mathematics is presented as low rather than high discursive saturation.

Insofar as they are representative of mathematics education more generally, the extracts from these two textbooks seem to give some justification to the position advanced in the ANC document quoted earlier. Divisions are being established within mathematical practices. We have, firstly, a division between esoteric domain and public domain practices. The esoteric domain comprises the principles of mathematical knowledge in discursive form. This is a formal domain in which mathematical objects are defined in terms of other mathematical objects. The public domain is the product of the projection of a mathematical gaze from the esoteric domain onto the world. Everyday practices are recontextualised so as to be describable in mathematical terms. The public domain thus constitutes a virtual reality. The esoteric domain exhibits high discursive saturation, however, its practices may be degraded and reproduced as facts and algorithms. In this form its principles are invisible, so that what is reproduced is a practice exhibiting low rather than high discursive saturation.

The targeting of the two textbooks on different categories of student effects a distribution of mathematical practices such that high discursive saturation esoteric domain practices are distributed to ‘high ability’ students and low discursive saturation public domain practices are distributed to ‘low ability’ students. The category ‘ability’ refers to competence. We can say, then, that the mathematics curriculum, differentiated in this way, measures student competence.

My further analysis of these texts suggests that a significant principle of recognition relating to ability is that of social class. For example, the differences in mode of presentation of the two series reflects the differentiation on format between the popular and serious press in the UK (Dowling, 1993a). Since there are fairly strong social class associations between these two newspaper formats (Tunstall, 1983), we can argue at least a correspondence between ‘ability’ and social class in the SMP scheme.

The scheme also distributes differential pedagogic practices to the different categories of student. The advice given in the Teacher’s Guides for the Y series makes it clear that there are specific mathematical messages to be transmitted:

> The method used for increasing an amount by 35%, for example, is to multiply by 1.35. Although more difficult to grasp, this method (and the corresponding method

> for percentage decreases) has distinct advantages over the more usual method in that it easily extends to such problems as 'what is the overall effect, in percentage terms, of two successive percentage increases of 30% and 35%?' or 'what amount, when increased by 15% becomes £250?'
> (Teacher's Guide to Book Y1, pp. 38-9)

What is important, here, is the introduction of a generalisable mathematical strategy and not the solution of any particular problem. The text is announcing the authority of mathematics which is voiced by the teacher:

> Many important points arise in the course of doing the problems in the books, and these points will need to be brought out by the teacher in discussion with the class.
> (ibid, pp. 9-10)

Discussion in the G texts takes on a rather different role:

> Discussion between pupils, and between pupil and teacher, is perhaps the most useful mathematical activity possible; 'talking through' with the teacher may be the only way to make the work relevant.
> (Teacher's Guide to Book G1, p. 8)

Here, the teacher is not voicing the authority of mathematics. On the contrary, relevance outside of mathematics is to be prioritised.

> We hope that much in the G materials will act as a 'model' for work of your own devising. work on timetables, map-reading, shopping and so on is far more motivating form pupils if it is seen to be 'real'. Blagdon
> can never substitute for your own town! So in a sense, we hope that some chapters in the books never get used by pupils. They are written to be replaced by work which is firmly based on the pupils' own environment. Of course, replacement may not always be possible, but work based on the pupils' own school, town or surroundings may be added to a particular chapter.
> (ibid; my footnote)

The 'generalising strategies' of the Y scheme are replaced by 'localising strategies' in the G books. Furthermore, the Y students are commonly construed as incompetent in relation to mathematical practices:

> Depending on the level of confidence of the class and the extent of their previous experience of investigations, it may be a good idea to introduce the first problem in chapter 8 (Cutting a cake) before reaching the chapter, and without any of the assistance given in the chapter. Those who try to solve the problem and 'get lost' are likely to appreciate more the need for a methodical approach.
> (Teacher's Guide to Book Y1, p. 10)

The G books, on the other hand, frequently omit any instruction on how to approach apparently everyday tasks:

> Pupils may be asked to explain how they solved a particular problem, and the different methods used by pupils can then be compared. Often for these pupils, there is no single 'correct way' of doing things. Rather there is one method which suits a particular pupil best for a particular problem.

(Teacher's Guide to Book G1, p. 8)

There is a sense, then, in which the community—represented by the students in the class—is attributed a competence in the form of a reservoir of problem-solving strategies which pre-exists the particular lesson. Rather than introducing strategies, pedagogy may facilitate the sharing of strategies from the community reservoir in the development of individual repertoires. It is always the case, however, that the public domain tasks themselves are recontextualised products of the mathematical gaze. Since the public domain is a 'virtual reality', there is no basis for the reservoir of competences. Furthermore, where strategies are introduced, their algorithmic nature can only obscure the principles of their origination within the esoteric domain. The G students are subjected by mathematical practices in that their competence is evaluated on the basis of invisible principles.

The pedagogic strategies of the Y texts, on the other hand, are more likely to reveal the principles of the esoteric domain. Y students are, potentially, subjects of mathematics. However, they are no more likely to generate plausible solutions to everyday practical problems, because the everyday is not structured according to mathematical principles, as has been argued earlier. It would seem, then, that the education which is represented by these SMP texts is geared to the production of an idle and impractical elite, on the one hand, and an untrained workforce, on the other.

On the face of it, then, there would seem to be some justification for a movement towards what Michael Young refers to as the 'curriculum of the future', towards the unification of theory and practice as advocated, last year by the ANC and now by the Department of Education in its recent White Paper (1995). The policy has a superficial appeal. But what precisely might be meant by the terms 'theory' and 'practice'? How might theory and practice be integrated and what might happen if they were? Perhaps these questions are comparatively easy to answer with respect to, say, the training of motor mechanics within trade apprenticeships. Certainly, there is some evidence of a disjunction between college curricula and on-the-job training in such schemes (see Lave & Wenger, 1991; also Shell Centre, nd, and discussion in Dowling, 1989). But is the situation the same in the case of teacher education or the training of medical practitioners? Can we interpret academic physics as a theory for something else? How about sociology? The illustrations above suggest that mathematics cannot be interpreted as a theory for domestic and other everyday practices. I want to argue that, similarly, educational research is not appropriately interpreted as generating theories for classroom practice. Rather, mathematics and educational research are practices in their own right. But they are practices of a particular kind. Essentially, calls for the integration of theory and practice ignore the social conditions of possibility of precisely this particular kind of practice.

Towards a classification of pedagogic types

In the previous section, I have introduced some of the terms of a language of description which I have developed for the sociological analysis of pedagogic texts. Crucial distinctions have been made between esoteric and public domains and between high and low discursive saturation. I want to contend that all practices generate specialised forms of practice, so that there is a sense in which all practices constitute a form of esoteric domain. This is true of domestic and everyday and working practices as well as academic activities, such as mathematics and sociology. However, the nature of the esoteric domains in terms of discursive saturation is contingent upon the structure of the pedagogic relations within which the practices are acquired. In this section of the paper, I want to introduce an ideal-typical schema of pedagogic relations which will enable me to argue the radical incommensurability of spheres of practice such as mathematics and the everyday or such as educational research and practice. This position calls into serious question any naive understanding of the utility of school mathematics for the everyday or of educational research for teachers' practices. The schema will also enable me to suggest the form of a productive relationship between such spheres.

The method of ideal types was introduced by Max Weber who used it to generate categories of, for example, social action (instrumental- and value-oriented) and authority (traditional, charismatic, bureaucratic) (see Weber, 1964). Ideal types are categories which originate from observation, but which have been made conceptually coherent. In referring back to the empirical, it is important to remember that concrete instances are likely to combine elements of more than one ideal type, so that the latter are unlikely to be found in their pure form. It is also important to stress that the construction of ideal types is a theoretical exercise, albeit with some grounding in observation; as Blau and Meyer (1971) point out, the method of ideal types is intended to be a guide to empirical research, not a substitute for it. Having announced these caveats, I shall proceed to describe my ideal types of pedagogic relations.

I want to start at the highest point of formal education, which is the supervision of PhD theses. Within this site, the supervisor is (or certainly should be) an expert practitioner in the pedagogic content—the methods and criteria of evaluation of academic research in the specific field of the thesis. The student acquires the content through engagement, in one form or another, with that expertise. The relation between supervisor and research student can thus properly be described as one of apprenticeship. That is, the goal of pedagogic action is to enable the student to become a subject of the field of expertise of the teacher. Achievement of this goal may be recognised at the point at which the student's performances can themselves stand as expert products within the field. These performances are likely to include papers presented at academic seminars and conferences and, possibly, also for publication in the journals. In this respect, the focus of the evaluation is on these performances as performances. The thesis, in particular, must stand, in its own right, as an original contribution to knowledge in the relevant field. It is not possible to compensate for a weak thesis on the grounds of, for example, inadequate supervision or illness or other unfortunate circumstances relating to the student; there are no aegrotat awards at doctoral level.

As I have argued in my paper, ‘Apprenticeship and Educational Research: a mode of interrogation’ (elsewhere in these proceedings), the mode of evaluation of these performances is discursive and the practice of evaluation itself forms part of the productive process itself: new work always arises out of the explicit evaluation and development of other work. Within this site, then, what is produced are products which are, in a sense, alienated from the producer, who claims authorship, but not ownership of them. These products—theses, research papers and monographs, and so on—are incorporated into discursive relationships with the products of other producers within the field. An academic product is always both complete and incomplete. Complete in the sense that it must satisfy criteria of rational coherence and that it must make some kind of statement. Incomplete in the sense that it always participates in the production of other academic products which always have more or less extensive bibliographies. To be a producer in the field, then, entails that one must become familiar with an intertextual terrain and with a specific mode of discursive interrogation.

The mode of interrogation constitutes the ways in which practitioners in the field engage with each other’s discursive performances. The supervisor and the examiners of the thesis—and, generally the student themself—are all participants in the field within their particular discipline. They are accountable within the discipline for their own output and for their interrogation of that of others, including PhD theses. In other words, these agents stand in discursive relation to each other, their own positions being marked by acknowledged authorship of publicly approved performances. The esoteric domains of the various disciplines must be described as high discursive saturation which are, therefore, capable of generating utterances the meanings of which are, to a substantial extent, independent of the immediate context of their production. Such esoteric domains are capable of generating systematic languages of description, such as those of mathematics and sociology. The pedagogic relation characterised by apprenticeship, focus on performance, and high discursive saturation, I shall refer to as the research type.

Moving to the lowest point in formal education, to the primary school, we find a somewhat different situation. Here, the teacher is, in general, no longer an expert practitioner in the pedagogic content. I am not suggesting that, for example, primary teachers teaching mathematics are not mathematically competent at the level at which they are teaching. Rather, that they will generally practice mathematics only in the context of teaching it and that their own mathematical competence is unlikely to extend very much higher than school level and almost never above first degree level. Primary school teachers, then, are not mathematicians, they are teachers, but they are not teaching their students to be teachers, rather, they are relaying another practice—mathematics. There is a sense, then, in which mathematics is constituted as a virtual practice—a public domain—of educational practice. It is recontextualised mathematics. It is fragmented and sequenced. It may be articulated with a recontextualised developmental psychology which may principle the curricular hierarchy to a greater or lesser extent. School mathematics is thus constituted as a scale against which the student is to be measured.

This pedagogic relationship is not one of apprenticeship. Students are not to become subjects of the practice in which their teachers are adepts. Rather, they are objectified by the curriculum. Student's performances are of no value in themselves, rather they are taken as indicators of something else which is, ultimately, the student's level of development which, in association with their chronological age, measures 'ability'. Higher up in the school system, even public examination performances are reduced to grades which certify the suitability of the holder for further study or for employment; the examination scripts themselves are ultimately destroyed. School workbooks may also be destroyed or they may be kept along with other mementos in personal archives, but for sentimental value rather than for their contribution to mathematics. Students are identified rather than subjectified by school mathematics. Schooling, conceived in this way, constitutes not subjects, but identities.

The principles of this objectification are, like those of the research mode of interrogation, available within discourse. They are institutionalised in curricula and syllabuses as well as in published textbooks and they are debated in professional journals and in state committees and publications. In addition, they generally stand in some discursive relationship to the disciplined knowledge of the research type. As with research, this type of pedagogic relation—the school type—is to be described as high in terms of discursive saturation, utterances have a relatively high degree of context-independence. However, some qualification is needed to this description. Essentially, the discursive regulation of the pedagogic relation is confined to the pedagogic content and not to pedagogic strategies in general: school mathematics is discursively regulated to a far greater extent than teachers' classroom techniques. Pedagogic practice as distinct from pedagogic content (ultimately, only an analytic distinction) may be more appropriately described as exhibiting relatively low discursive saturation. This is also the case in the research type, but there, that which is to be interrogated—the performance—is available for public monitoring. In the school, individual 'abilities' cannot be discursively available in the same way. Furthermore, it is precisely in the area of pedagogic strategies that teachers' professional expertise can be said to lie. Therefore, the lack of languages of description for the interrogation of their practice may be considered to be a problem. I will return to this point at the end of the paper.

The third pedagogic type is that of the family, the domestic type. As is the case in the school, there is the establishing of virtual practices. The parent is, generally, not teaching the child how to be a parent, but how to be a child. To this extent, primary socialisation within the family is not apprenticeship, although there may be elements of apprenticing within empirical family settings. The virtual practices that constitute childhood are recontextualisings from what are certainly diverse sources: the parent's own childhood; images in the media; direct and vicarious experiences of other families. As in the school, the focus of evaluation is on identifying the child: the child's performances indicate what kind of a child they are and the nature of their individual needs. Parents become experts in terms of knowledge about their children who are thereby constituted as individualised objects. The regulation of the objectification is not constituted by a discursive field, and this marks out domestic pedagogic relations from those obtaining in the research and school types. Primary socialisation in the family exhibits low discursive saturation.

Finally, I turn to traditional craft apprenticeship—the craft type. Here, the dominant figure in the pedagogic relationship is, like the PhD supervisor, an adept in the practice that comprises the pedagogic content. The novice's performances, like those of the PhD student, are submitted to a mode of interrogation which, ultimately, coincides with that to which the adept's own performances are submitted. For example, Singleton (1989) reports that an apprentice in a Japanese mingei pottery is often told to make ten thousand small sake drinking cups exactly corresponding to the masters' [sic] model. Initially, none of the cups will be fired, but will be returned to the clay pit for recycling.

> An apprentice may spend six months or more making the first simple shape at the wheel, after his other chores have been attended to, not actually counting to see if the goal of ten thousand has been reached. When the potter begins to select some of the cups for firing, to be finished and sold as unsigned products of the shop, the apprentice has moved from practice to production. It is an important change of status, though there are still other forms to practice and master. (Singleton, 1989; p. 20)

However, these performances are not implicated in discursive relations between potters or between potters and apprentices. Relations are essentially economic rather than discursive. In the case of mingei pottery, the acquisition of the skills is expected to proceed through observation and practice and without any didactic instruction (Singleton, 1989). However, even where direct instruction is given, it is unlikely to be discursively regulated and utterances will be highly context-dependent. This is because the principles of interrogation are not available within a discursive field. The craft type of pedagogic relations is constituted by and within a practice exhibiting low discursive saturation.

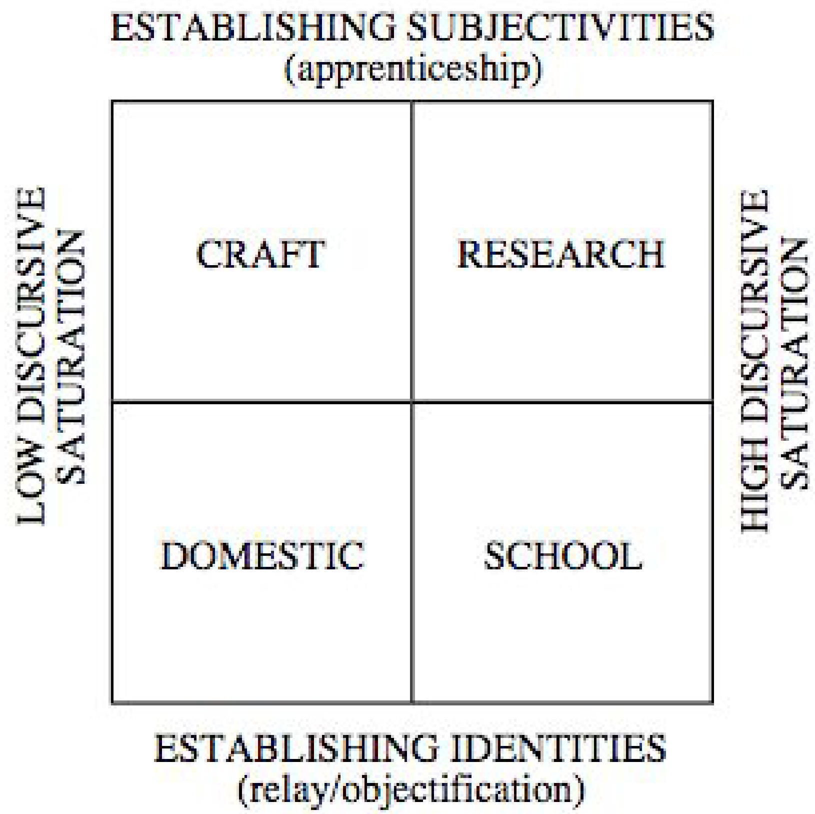

Figure 1: Ideal typical schema of pedagogic relations

The four ideal types can be arranged diagrammatically, as in Figure 1. As I have already indicated, empirical instances of pedagogic relations are likely to involve combinations of these types. Craft apprenticeships in the UK and in the US, for example, often include college-based elements, so that the apprenticeship as a whole might be described as a combination of craft and school types. It is commonly found, however, that the two components are kept very separate from each other and the formal component is often regarded as generally irrelevant to the real job, as I indicated earlier. Aspects of domestic pedagogy may very well look more like apprenticeship. This might be the case in the acquisition of cooking and shopping skills. In my description of the research and school types, I have focused attention on the two extreme points of the formal education system. Between these two points, we might describe a transition. First degree teaching may have as much in common with the secondary school as it does with PhD supervision. As I have illustrated with respect to the SMP texts, pedagogic relations within the secondary school may also be differentiated according to ‘ability’, recognised in terms of social class. Only for ‘higher ability’/middle class students is the authority of the discipline emphasised, shifting the focus of evaluation somewhat towards performance.

Of particular interest is the training of school teachers in the UK. Secondary school teachers, in particular, will generally take a first degree in an academic subject such as mathematics or history. They will then proceed to a one-year full-time Postgraduate Certificate in Education (PGCE) course. The PGCE is generally managed and certified by an institution of higher education. However, the state regulates the content of the courses which include very little academic work. Furthermore, state intervention has shifted much of the teaching on the courses into schools themselves. School teachers have also been given increasing responsibility for the delivery and assessment of the practical aspects of the course. This is a move towards an apprenticeship model. As I have argued, however, the practice of teaching as teaching can appropriately be described as low discursive saturation. In terms of the ideal typical schema, we can describe PGCE pedagogic relations as having shifted from research to craft.

The reduction in profile of the research pedagogy in initial teacher education is mirrored in other developments in respect of educational research and higher degree teaching. These include the increasing emphasis on practice-oriented research at the expense of fundamental research; the tendency for institutions to offer ad hoc masters courses on the basis of demand (often from 'Third World' states and funded by organisations such as the World Bank); and the moves towards a US model of PhD, which generally includes taught courses and has a less substantial thesis. In relation to the latter move, some institutions are now considering the introduction of the faculty-specific Doctor of Education (EdD) degree. This award is of a lower status than the PhD and is often intended to be geared towards curriculum development or other professionally-oriented work rather than what is traditionally regarded as academic research.

These shifts in higher education may be interpreted as being associated with the general hegemony of economisation in the UK and elsewhere. This process has generated, in the UK, the introduction of an objectives-based National Curriculum, national assessment, a unified examination system and the publication of league tables of school performances. These innovations may be seen as attempts to transform discursive relations into market relations through the construction of an economy of school performances. This clearly militates against the development of academic practices exhibiting high discursive saturation. At the same time, there is a general trend towards vocationalism. This is represented by the extract from the ANC document quoted earlier, but also characterises many developments in the UK and elsewhere. Insofar as such a move is successful, then it must effect an increase in the context-dependency of pedagogic texts and an associated reduction in the level of discursive saturation of the practice. On the other hand, we might speculate that the move will not be successful, so that pedagogic texts remain substantially dissociated from everyday and working practices, rather like the college courses accompanying trade apprenticeships. To the extent that such courses are economised via their regulation by curricula defined in terms of objectives, however, there will still be a denaturing of the discursivity of pedagogic content.

Conclusion: towards a productive relationship between domains

In this paper, I have taken it as axiomatic that all practices generate more or less specialised sets of practices, esoteric domains. Certain forms of practice—those that I have described as exhibiting low discursive saturation—regulate their esoteric domains via context-dependency. You have to be in the supermarket in order to do the shopping and doing the shopping is never quite the same as writing out the shopping list. Doing the shopping is always very different from solving a problem in a mathematics textbook.

Those practices which display high discursive saturation regulate their esoteric domains discursively and are far less context-dependent. These practices, such as mathematics and sociology, are able to generate highly systematic languages of description. However, in their descriptions, these practices constitute the world as public domains of their own expression. Mathematics constitutes shopping as a mathematically rational practice; sociology constitutes teachers' classroom activities as sociologically coherent. The effect is always a recontextualising—a mythologising—of practice. Mathematised shopping is not shopping; sociologised teaching is not teaching.

I have argued, further, that the potential to develop languages of description is contingent upon the performances of a practice being implicated in discursive relations. Only the research type of pedagogic relations is predicated upon students' involvement in such relations. The increasing marginalisation of these relations within the sphere of education, their relegation to a Twilight Zone, must lead to a reduction in the potential for the generation of languages of description with an educational focus. Similarly, attempts to vocationalise mathematics and science education must also decrease access on the part of students—and, ultimately, teachers as well—to the esoteric domain of mathematical knowledge. But if sociologised teaching is not teaching and mathematised shopping is not shopping, why should this matter?

Essentially, to the extent that it is coherent, no practice can interrogate itself. Where practices are context-dependent, things are the way they are because that's the way they are. Abstractions are undisciplined and so easily dismissed as whimsical. Systematically organised languages, on the other hand, facilitate disciplined interrogation of practices. On the other hand—and just as crucially—the empirical discipline of practices exhibiting low discursive saturation must provide a grounding for potentially untrammelled theorising: discursively self-referential practices can otherwise rationalise any position. The relationship between the two modes of practice is dialogic.

The kind of dialogue that I am describing is exemplified in developments in relation to language in present-day South Africa. On the one hand, the fact that most South Africans routinely speak languages rather than a language is clearly likely to have the effect of blurring the distinctiveness of individual languages— in terms of their lexical and grammatical structures—as they are spoken in everyday life. On the other hand, the moves to install languages other than English and Afrikaans on the school curriculum will result in the production of textbooks, grammars, dictionaries and syllabuses which will tend to institutionalise standardised forms of these languages, forms which mythologise actual language use. The result is, I suggest, a productive tension rather than a contradiction. A tension that will not and ought not be resolved. The situation is comparable in relation to low discursive saturation and high discursive saturation practices more generally. They must retain, to a degree, their respective integrities, but they must not operate in isolation.

A potential objection to my position is that there are hierarchies between the kinds of practice that I have been describing. The mythologising of research and of mathematics is often quite rightly interpreted as moralising: this is the way you should teach; this is the way you should do your shopping. In South Africa, this is also—and again correctly—interpreted as a moralising of Africans by Europeans. Even those who are clearly striving for emancipation, such as the ethnomathematicians end up projecting European high culture onto the cultural practices of those whom they wish to serve (see Dowling, 1993b, in press).

However, to respond by rejecting the academic is to attempt to resolve problems in the social division of labour through cultural tinkering. It is true that dominant groups have appropriated high discursive saturation practices, but they have also appropriated low discursive saturation practices in the wider economy. Here, that which is appropriated is generally referred to as property. Social inequality is sustained through both economic and discursive means. It is the fundamental nature of the division of labour that must be addressed, not cultural expressions. That fundamental nature is a division which operates between human subjects. Humans are workers or they are bosses; Africans or Europeans; artisans or mathematicians; teachers or educational researchers.

Reorganising these divisions so that they penetrate rather than divide individuals is no small task. But mathematics teachers can at least stop pretending that they are teaching mathematics because you need it to do the shopping properly. Instead, they can introduce their students to the esoteric domain of mathematics to enable them to become the subjects of its languages. When we're doing maths we should suspend our concerns about its practical utility. Academics can stop designing courses on how to assess the school mathematics curriculum. Instead, they can encourage teachers to engage in fundamental research and, perhaps, engage in more of it themselves. When we're doing research, we should stop worrying about how it's going to help us out in the classroom on Monday morning. The Twilight Zones of mathematics and of research are not zones of proximal development attached to the supermarket or to the classroom. In fact, they become magically illuminated when we enter them, leaving our preconceptions behind in what now become the Twilight Zones of the everyday and the school.

The voice of the mathematics teacher, the voice of the educational researcher will always be voices from the Twilight Zone so long as they confine their discourse to other people's practices. These voices will always remain strange aberrations so long as curriculum developers insist that they speak in a language other than their own. The point is not to prescribe practices for others, nor to demand prescriptions from others, but to give and acquire access to the resources for self-description.

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The book is part of a scheme published by Cambridge University Press. This is currently the most popular secondary mathematics scheme in England and Wales.

The notion of an irrational number does not appear explicitly in the Y series. The irrational nature of π is implied by the use of approximations rather than an exact value and by the expression of its decimal expansion to thirty-five places of decimals on what appears to be a very long strip of paper which curls round after the thirty-fifth place (concealing the subsequent digits) and snakes off the edge of the page.

There is also one esoteric domain task.

The name of an apparently fictional town referred to frequently in the SMP texts. There is no obvious reference to Blagdon in Avon or Blagdon in Devon.

The expression, 'language of description' originated with Basil Bernstein whose work has significantly influenced my own (see, for example, Bernstein, 1990; Dowling, 1993a, 1994).

The danger of a descent into theoreticism which is immanent in the ignoring of these methodological issues is illustrated by Hunter (1994) who reifies Weber's 'bureaucratic authority' in his positivist critique of liberal and radical accounts of schooling (see Dowling, 1994/5).

Both of these disciplines are described as high discursive saturation, although there are clearly some important differences. Principally, there is a far greater tendency for mathematical languages to be consistent with each other that is the case with sociology. Mathematical languages specialise, whilst sociological languages are in competition (Bernstein, private communication). Further, Mathematics is formalised to a far greater extent than sociology, so that there must be a far greater reliance on the canon and on exemplary texts in the latter.

As I noted earlier, empirical instances are likely to entail combinations of ideal types rather than represent pure forms.

See Coy (1989) for a collection of anthropological studies involving apprenticeship; see also the examples in Lave & Wenger, 1991.

Clearly, all processes of induction into professions involve a substantial element of the craft type of pedagogic relation. However, the teaching profession—at least in the UK—seems to be being driven further than others in the elimination of academic knowledge specific to its professional practice, that is, educational research. To the extent that teachers are complicit in this process, teaching may be described as a rare case of a profession disowning the intellectual basis of its specialisation.

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